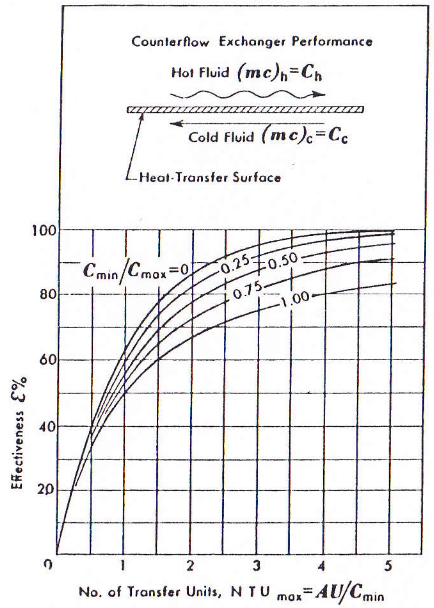

FIG. 11-17. Heat-exchanger effectiveness for counterflow. (By permission from W. M. Kays and A. L. London, *Compact Heat Exchangers*, National Press, 1955)

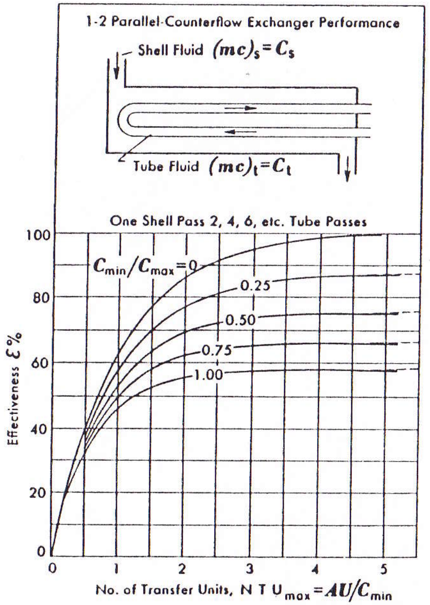

FIG. 11-18. Heat-exchanger effectiveness for shell-and-tube heat exchanger with one well-baffled shell pass and two, or a multiple of two, tube passes. (By permission from W. M. Kays and A. L. London, *Compact Heat Exchangers*, National Press, 1955)

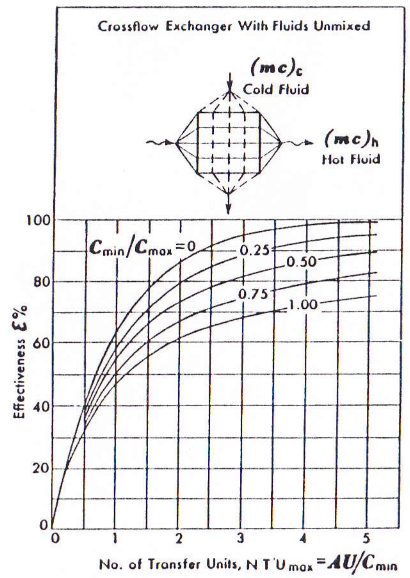

FIG. 11-19. Heat-exchanger effectiveness for crossflow with both fluids unmixed. (By permission from W. M. Kays and A. L. London, *Compact Heat Exchangers*, National Press, 1955)

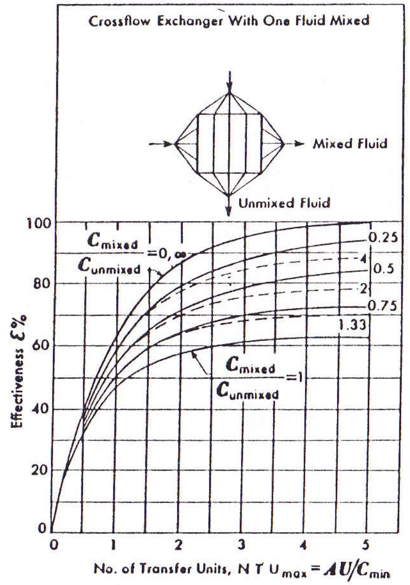

FIG. 11-20. Heat-exchanger effectiveness for crossflow with one fluid mixed, the other unmixed. When $C_{mixed}/C_{unmixed} > 1$, NTU_{max} is based on $C_{unmixed}$. (By permission from W. M. Kays and A. L. London, *Compact Heat Exchangers*, National Press, 1955)

heat capacity, and the difference between the inlet temperatures. It re-
places Eq. 11–12 in the LMTD analysis but does not involve the outlet
temperatures. Equation 11–18 is of course also suitable for design pur-
poses instead of Eq. 11–12.

We shall illustrate the method of deriving an expression for the ef-
fectiveness of a heat exchanger by applying it to a parallel-flow arrange-
ment. The effectiveness can be introduced into Eq. 11–8 by replacing
$(T_{c\ in} - T_{c\ out})/(T_{h\ in} - T_{c\ in})$ by the effectiveness relation from Eq. 11–17.
We obtain

$$\ln\left[1 - \varepsilon\left(\frac{C_{min}}{C_h} + \frac{C_{min}}{C_c}\right)\right] = -\left(\frac{1}{C_c} + \frac{1}{C_h}\right)UA$$

or

$$1 - \varepsilon\left(\frac{C_{min}}{C_h} + \frac{C_{min}}{C_c}\right) = e^{-(1/C_c + 1/C_h)UA}$$

Solving for ε yields

$$\varepsilon = \frac{1 - e^{-[1+(C_h/C_c)]UA/C_h}}{(C_{min}/C_h) + (C_{min}/C_c)} \qquad \text{(11–19)}$$

When C_h is less than C_c, the effectiveness becomes

$$\varepsilon = \frac{1 - e^{-[1+(C_h/C_c)]UA/C_h}}{1 + (C_h/C_c)} \qquad \text{(11–20)}$$

and when $C_c < C_h$, we obtain

$$\varepsilon = \frac{1 - e^{-[1+(C_c/C_h)]UA/C_c}}{1 + (C_c/C_h)} \qquad \text{(11–20a)}$$

The effectiveness for both cases can therefore be written in the form

$$\varepsilon = \frac{1 - e^{-[1+(C_{min}/C_{max})]UA/C_{min}}}{1 + (C_{min}/C_{max})} \qquad \text{(11–21)}$$

The foregoing derivation illustrates how the effectiveness for a given flow
arrangement can be expressed in terms of two dimensionless parameters, the
hourly heat-capacity ratio C_{min}/C_{max} and the ratio of the over-all conduct-
ance to the smaller hourly heat capacity, UA/C_{min}. The latter of the two
parameters is called the *number of heat-transfer units*, or NTU for short. The
number of heat-transfer units is a measure of the heat-transfer size of the
exchanger. The larger the value of NTU, the closer the heat exchanger
approaches its thermodynamic limit. By analyses which in principle are
similar to the one presented here for parallel flow, effectivenesses may be
evaluated for most flow arrangements of practical interest. The results
have been put by Kays and London (1) into convenient graphs from which
the effectiveness can be determined for given values of NTU and C_{min}/C_{max}.

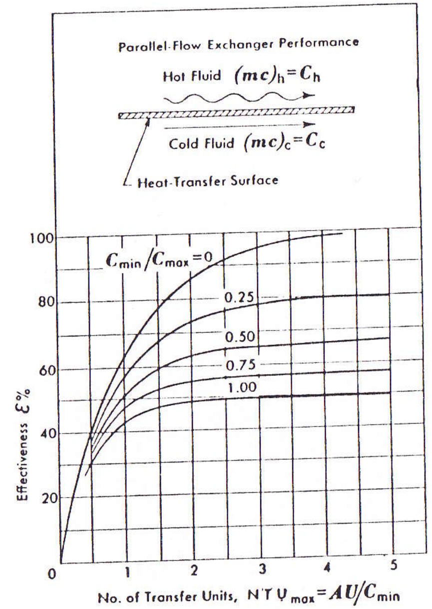

FIG. 11–16. Heat-exchanger effectiveness for parallel flow. (By permission from W. M. Kays and A. L. London, *Compact Heat Exchangers*, National Press, 1955)

The effectiveness curves for some common flow arrangements are shown in Figs. 11–16 to 11–20. The abscissas of these figures are the NTU's of the heat exchangers. The constant parameter for each curve is the hourly heat capacity ratio C_{min}/C_{max}, and the effectiveness is read on the ordinate. Note that, for an evaporator or condenser, $C_{min}/C_{max} = 0$, because if one fluid remains at constant temperature throughout the exchanger, its effective specific heat, and thus its capacity rate, is by definition equal to infinity.

Example 11–2. From a performance test on a well-baffled single-shell, two-tube-pass heat exchanger, the following data are available: oil ($c_p = 0.5$ Btu/lb F) in turbulent flow inside the tubes entered at 160 F at the rate of 5000 lb/hr and left at 100 F; water flowing on the shell side entered at 60 F and left at 80 F. A change in service conditions requires the cooling of a similar oil from an initial temperature of 200 F but at three fourths of the flow rate used in the performance test. Estimate the outlet temperature of the oil for the same water rate and inlet temperature as before.

Solution: The test data may be used to determine the hourly heat capacity of the water and the over-all conductance of the exchanger. The hourly heat capacity of the water is from Eq. 11–9

$$C_c = C_h \frac{T_{h\ in} - T_{h\ out}}{T_{c\ out} - T_{c\ in}} = (5000)(0.5)\left(\frac{160 - 100}{80 - 60}\right) = 7500 \text{ Btu/hr F}$$